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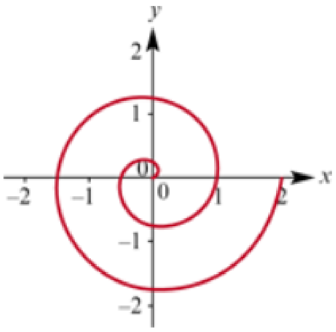


2 **a** Since $x = r\cos\theta$, this equation becomes

$$\begin{aligned} x &= 4 \\ r\cos\theta &= 4 \\ r &= \frac{4}{\cos\theta} \end{aligned}$$

b Since $x = r\cos\theta$ and $y = r\sin\theta$ this equation becomes

$$\begin{aligned} xy &= 1 \\ r^2\cos\theta\sin\theta &= 1 \\ r^2 &= \frac{1}{\cos\theta\sin\theta} \end{aligned}$$

c Since $x = r\cos\theta$ and $y = r\sin\theta$ this equation becomes

$$\begin{aligned} y &= x^2 \\ r\sin\theta &= r^2\cos^2\theta \\ r\cos^2\theta &= \sin\theta \\ r &= \frac{\sin\theta}{\cos^2\theta} \\ &= \tan\theta\sec\theta \end{aligned}$$

d This is just a circle of radius 3 centred at the origin and so has equation $r = 3$. We can check this by letting $x = r\cos\theta$ and $y = r\sin\theta$ this equation becomes

$$\begin{aligned} x^2 + y^2 &= 9 \\ r^2\cos^2\theta + r^2\sin^2\theta &= 9 \\ r^2(\cos^2\theta + \sin^2\theta) &= 9 \\ r^2 &= 9 \end{aligned}$$

In fact, since we are allowing negative r values we could take either $r = 3$ or $r = -3$ as the equation of this circle.

e Since $x = r\cos\theta$ and $y = r\sin\theta$ this equation becomes

$$\begin{aligned} x^2 - y^2 &= 9 \\ r^2\cos^2\theta - r^2\sin^2\theta &= 1 \\ r^2(\cos^2\theta - \sin^2\theta) &= 1 \\ r^2\cos 2\theta &= 1 \\ r^2 &= \frac{1}{\cos 2\theta} \end{aligned}$$

f Since $x = r\cos\theta$ and $y = r\sin\theta$ this equation becomes

$$\begin{aligned} 2x - 3y &= 5 \\ 2r\cos\theta - 3r\sin\theta &= 5 \\ r(2\cos\theta - 3\sin\theta) &= 5 \\ r &= \frac{5}{2\cos\theta - 3\sin\theta} \end{aligned}$$

3 a The trick here is to first multiply both sides of the expression through by $\cos\theta$ to get

$$r\cos\theta = 2$$

Since $r\cos\theta = x$, this equation simply becomes,

$$x = 2.$$

b Since $r = 2$, this is just a circle of radius 2 centred at the origin. Its cartesian equation will then be simply

$$x^2 + y^2 = 2^2.$$

c Here, for all values of r the angle is constant and equal to $\pi/4$. This corresponds to the straight line through the origin, $y = x$. To see this algebraically, note that

$$\frac{y}{x} = \tan(\pi/4) = 1.$$

Therefore, $y = x$.

d Rearranging the equation we find that

$$\begin{aligned} \frac{4}{3\cos\theta - 2\sin\theta} &= r \\ r(3\cos\theta - 2\sin\theta) &= 4 \\ 3r\cos\theta - 2r\sin\theta &= 4 \quad (1) \end{aligned}$$

Then since $x = r\cos\theta$ and $y = r\sin\theta$, equation (1) becomes

$$3x - 2y = 4.$$

4 a The trick here is to first multiply both sides of the expression through by r to get

$$r^2 = 6r\cos\theta \qquad (1)$$

Since $r^2 = x^2 + y^2$ and $r\cos\theta = x$, equation (1) becomes,

$$\begin{aligned} x^2 + y^2 &= 6x \\ x^2 - 6x + y^2 &= 0 \\ \text{(completing the square)} \\ (x^2 - 6x + 9) - 9 + y^2 &= 0 \\ (x-3)^2 + y^2 &= 9. \end{aligned}$$

This is a circles whose centre is $(3, 0)$ and whose radius is 3.

b The trick here is to first multiply both sides of the expression through by r to get

$$r^2 = 4r\sin\theta \qquad (1)$$

Since $r^2 = x^2 + y^2$ and $r\sin\theta = y$, equation (1) becomes,

$$\begin{aligned} x^2 + y^2 &= 4y \\ x^2 + y^2 - 4y &= 0 \\ \text{(completing the square)} \\ x^2 + (y^2 - 4y + 4) - 4 &= 0 \\ x^2 + (y-2)^2 &= 4. \end{aligned}$$

This is a circle whose centre is $(0, 2)$ and whose radius is 2.

c The trick here is to first multiply both sides of the expression through by r to get

$$r^2 = 2r\sin\theta \qquad (1)$$

Since $r^2 = x^2 + y^2$ and $r\cos\theta = x$, equation (1) becomes,

$$\begin{aligned} x^2 + y^2 &= -6x \\ x^2 + 6x + y^2 &= 0 \\ \text{(completing the square)} \\ (x^2 + 6x + 9) - 9 + y^2 &= 0 \\ (x+3)^2 + y^2 &= 9 \end{aligned}$$

This is a circle whose centre is $(-3, 0)$ and whose radius is 3.

d The trick here is to first multiply both sides of the expression through by r to get

$$r^2 = 2r\sin\theta \qquad (1)$$

Since $r^2 = x^2 + y^2$ and $r\sin\theta = y$, equation (1) becomes,

$$x^2 + y^2 = -8y$$
$$x^2 + y^2 + 8y = 0$$
$$x^2 + (y^2 + 8y + 16) - 16 = 0$$
$$\text{(completing the square)}$$
$$x^2 + (y+4)^2 = 16.$$

This is a circle whose centre is $(0, -4)$ and whose radius is 4.

5 The trick here is to first multiply both sides of the expression through by r to get

$$r^2 = 2ar\cos\theta \qquad (1)$$

Since $r^2 = x^2 + y^2$ and $r\cos\theta = x$, equation (1) becomes,

$$x^2 + y^2 = 2ax$$
$$x^2 - 2ax + y^2 = 0$$
$$(x^2 - 2ax + a^2) - a^2 + y^2 = 0$$
$$\text{(completing the square)}$$
$$(x-a)^2 + y^2 = a^2.$$

This is a circle whose centre is $(a, 0)$ and whose radius is a.

6 a The trick here is to first multiply both sides of the expression through by $\cos\theta$ to obtain,

$$r\cos\theta = a$$
$$x = a,$$

which is the equation of a vertical line.

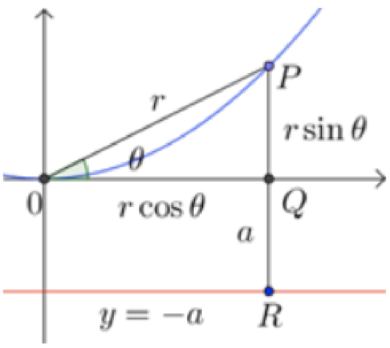
b Let $y = r\sin\theta$ so that

$$r\sin\theta = a$$
$$r = \frac{a}{\sin\theta}.$$

7 a The distance from P to the line is

$$RP = RQ + QP$$
$$= a + r\sin\theta.$$

b Consider the complete diagram shown below.



Since we are told that $OP = RP$, this implies that

$$OP = RP$$
$$r = a + r\sin\theta$$
$$r - r\sin\theta = a$$
$$r(1 - \sin\theta) = a$$
$$r = \frac{a}{1 - \sin\theta},$$

as required.